The power spectrum of the atomic dipole moment

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This paper calculates the power spectrum $S(\omega)$ of the electric field generated by the atomic dipole moment of a laser-driven two-level system from an open quantum systems perspective. Its shape is similar to the shape of Mollow's resonance fluorescence spectrum but there are some differences. For sufficiently strong laser driving, there are two sidebands but their relative height is reduced. Moreover, the amplitude of this spectrum has a different dependence on the laser Rabi frequency Ω. It does not vanish when Ω tends to zero. The calculation of the spectrum which we present here involves less approximations than the calculation of Mollow's spectrum and constitutes an interesting alternative property.

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I. INTRODUCTION

Recent years have seen a large number of experiments which study light-matter interactions at the quantum level using trapped ions [1], single quantum dots [2], color centres [3], and molecules on surfaces [4]. What all of these systems have in common is that they enable the excitation of a single trapped electron into a well-defined excited state which can result in the spontaneous emission of a photon. So-called artificial atoms, like quantum dots and color centres, often have much stronger spontaneous decay rates Γ and much smaller transition frequencies ω_0, thereby providing a new testing ground for quantum optical standard approximations.

Moreover, recent technological advances [5] opened new possibilities in measuring extremely small forces. For example, Usenko *et al.* [6] recently demonstrated the detection of subattonewton forces at millikelvin temperatures by using a superconducting quantum interference device. Other authors employ quantum point contacts as sensitive displacement detectors in high precision experiments which aim at quantum limited displacement detection [7, 8, 10]. The technology needed to combine a single quantum dot and an atomic force microscope cantilever in a single experimental setup has already become available [11].

Motivated by these developments, this paper calculates the power spectrum $S(\omega)$ of the electric field generated by a laser-driven atomic two-level system. We theoretically analyse the signal $f(t)$ which is given by the expectation value $\langle \mathbf{E}(\mathbf{x})\rangle_t$ of the electric field generated at a time t at a detector position $\mathbf{x}$. Due to constant environmental resetting, the free radiation field contains on average less than a single photonic excitation [12–15]. Since single photons have a zero electric field expectation value, the only contribution to the signal $f(t)$ comes from the atomic dipole moment $\mathbf{r}$. Since the atomic dipole moment of an atomic two-level system is essentially given by the Pauli operator $\sigma_{\rm x} = \sigma^+ + \sigma^-$, we find that

$$f(t) \;=\; \langle \sigma_{\rm x} \rangle_t \tag{1}$$

up to an overall factor which does not depend on the atomic dynamics. As we shall see below, the power spectrum $S(\omega)$ associated with the signal in Eq. (1) is a direct measure for the coherence of the atomic source. Like the Mollow spectrum, it exhibits three peaks with the central peak being at $\omega = \omega_0$. The main difference is a significant reduction of the relative height of the sidebands and a different dependence of the amplitude of $S(\omega)$ on the laser Rabi frequency Ω.

The power spectrum of the signal $f(t) = \langle \mathbf{E}(\mathbf{x})\rangle_t$ provides a direct measure of the atomic dynamics, but its derivation does not involve as many approximations as are used in conventional derivations of Mollow's fluorescence triplet [16–19]. For example, we treat the atomic system and the surrounding free radiation field as an open quantum system and assume a continuous loss of memory between the atomic system and its environment [20]. The first measurements of the Mollow triplet were performed in the seventies [21–24]. Recently there has been a huge revival in this type of experiment. Quantum systems under investigation include atoms [25], single molecules [4], quantum dots [26, 27], and quantum dots inside an optical cavity [28, 29].

There are four sections in this paper. In Section II, we discuss the theoretical background for our calculations. We introduce the electric field operator $\mathbf{E}(\mathbf{x})$, summarize the atomic time evolution of a laser-driven atomic two-level system, and shortly review Mollow's spectrum. In Section we derive the power spectrum of the atomic dipole moment and compare its features with the properties of Mollow's spectrum. Finally, we summarise our findings in Section IV.

II. THEORETICAL BACKGROUND

Quantum optical systems with spontaneous photon emission usually need to be described as *open* quantum systems. For example, when analysing an atomic system coupled to a free radiation field, it is assumed that a photon-absorbing environment resets the free radiation field on a coarse grained time scale Δt onto its vacuum state such that, at any given time t, there is at most one

photon in the $\mathbf{k}\lambda$-modes of the free radiation field [12–15, 31]. The vacuum is the einselected state [30] of a free radiation field with a photon-absorbing environment. In the absence of a photon source, it is the only state which does not evolve in time. This makes it the preferred state into which the radiation field relaxes rapidly once a photon has been placed into it. Based on this assumption, the standard Markovian master equation for atomic systems can be derived without having to employ a wide range of approximations [31].

The most important of these approximations is the rotating wave approximation. It can be avoided by means of a unitary transformation producing a diagonal Hamiltonian for which the bare photon vacuum with the source (e.g. the atomic system) in its ground state is the true ground state of the total Hamiltonian [32]. Only within this rotating wave representation, continuous environmental resetting onto the photon vacuum yields a zero stationary state photon emission rate, regardless of whether or not the Markovian approximation of extending time integrals to infinity has been made [31]. Since it is physically reasonable to assume that the stationary state photon emission rate of the source be zero in the absence of any external driving we identify the vacuum of the rotating wave representation as the preferred photon vacuum onto which the environment resets. By expressing a physical observable in terms of the photon creation and annihilation operators of the rotating wave representation we can then determine the effect of continuous environmental reseting on its dynamics.

A. The electric field operator

The signal $f(t)$ which we analyse in this paper is the expectation value of the electric field scattered by a laser-driven atomic system. In the rotating wave representation the relevant observable $\mathbf{E}$ at a detector with position $\mathbf{x}$ is the sum of two components [31],

$$\begin{aligned} \mathbf{E}(\mathbf{x}) &= -\mathrm{i} \sum_{\mathbf{k}\lambda} \left(\frac{\hbar \epsilon_0 \omega_k}{2V} \right)^{1/2} \mathbf{e}_{\mathbf{k}\lambda} \, a_{\mathbf{k}\lambda} \, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \mathrm{H.c.} \\ & \quad -e \sum_{\mathbf{k}\lambda} \left(\frac{\hbar \epsilon_0 \omega_k}{2V} \right)^{1/2} \frac{\omega_k}{\omega_0 + \omega_k} \mathbf{e}_{\mathbf{k}\lambda} (\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{r}) \, , \end{aligned} \tag{2}$$

where $a_{\mathbf{k}\lambda}$ is the annihilation operator (in the rotating wave representation) for a single photon with wave vector $\mathbf{k}$, polarisation λ, and frequency ω_k, while ϵ_0 and V are the dielectric constant and the field quantisation volume. Since single photons have vanishing electric field expectation values all non-zero contributions to $\langle \mathbf{E}(\mathbf{x}) \rangle_t$ come from this second term in Eq. (2) provided we assume the environment continuously resets the radiation field into the rotating wave vacuum.

Neglecting an overall factor which depends only on parameters like the actual position $\mathbf{x}$ of the detector but not the state of the atomic system and taking into account that the atomic dipole moment $\mathbf{r}$ is proportional to $\sigma_{\mathrm{x}} \equiv \sigma^+ + \sigma^-$ in the Schrödinger picture, we arrive at Eq. (1). We remark that if overall factors are to be ignored, then using an expression for the electric field in another gauge, e.g. the Coulomb gauge, also results in Eq. (1) provided it is assumed that the radiation field is reset onto the photon vacuum belonging to that representation. Thus, when overall factors are ignored, the only nonzero contribution to the electric field expectation value is equal to the expectation value of the static longitudinal electric field. We emphasise however that if the environment continuously resets the radiation field into the rotating wave vacuum, then what would actually be measured is the component represented by the second term in Eq. (2) which is not merely the longitudinal electric field.

B. Atomic master equations

To calculate $\langle \sigma_{\mathrm{x}} \rangle_t$, we employ in the following the standard quantum optical master equation of a resonantly driven atomic two-level system with ground state $|1\rangle$, excited state $|2\rangle$ and spontaneous decay rate Γ,

$$\begin{aligned} \dot{\rho}_{\mathrm{A}}(t) &= -\frac{\mathrm{i}}{\hbar} \left[\hbar\omega_0 \, \sigma^+ \sigma^- + H_{\mathrm{L}}(t), \rho_{\mathrm{A}}(t) \right] \\ & \quad + \Gamma \, \sigma^- \, \rho_{\mathrm{A}}(t) \, \sigma^+ - \frac{1}{2} \Gamma \, \left\{ \sigma^+ \sigma^-, \rho_{\mathrm{A}}(t) \right\} \, , \\ H_{\mathrm{L}}(t) &= \frac{1}{2} \hbar \Omega \, \sigma^+ \, \mathrm{e}^{\mathrm{i}\omega_0 t} + \mathrm{H.c.} \end{aligned} \tag{3}$$

Here $\sigma^+ = |2\rangle\langle 1|$ and $\sigma^- = |1\rangle\langle 2|$ are the atomic lowering and raising operator, $\hbar\omega_0$ denotes the energy difference between the atomic levels, and Ω is a real laser Rabi frequency. Moving into the interaction picture with respect to

$$H_0 = \hbar\omega_0 \, \sigma^+ \sigma^- \, , \tag{4}$$

the master equation in Eq. (3) becomes time-independent and can be solved analytically. Doing so and using the notation $\rho_{ij} \equiv \langle i | \rho_{\mathrm{AI}} | j \rangle$ with $\rho_{22} = 1 - \rho_{11}$ and $\rho_{21} = \rho_{12}^*$, we obtain

$$\mathrm{Re} \, \rho_{12}(t) = \mathrm{e}^{-\Gamma t/4} \, \mathrm{Re} \, \rho_{12}(0) \, , \tag{5}$$

while,

$$\begin{aligned} & \begin{pmatrix} \rho_{11}(t) \\ \mathrm{Im} \, \rho_{12}(t) \end{pmatrix} \\ & \quad = \left[\cos\mu t - \frac{1}{4\mu} \begin{pmatrix} \Gamma & 4\Omega \\ -4\Omega & -\Gamma \end{pmatrix} \sin\mu t \right] \mathrm{e}^{-3\Gamma t/4} \\ & \qquad \begin{pmatrix} \rho_{11}(0) - \rho_{11}^{\mathrm{ss}} \\ \mathrm{Im} \, \rho_{12}(0) - \mathrm{Im} \, \rho_{12}^{\mathrm{ss}} \end{pmatrix} + \begin{pmatrix} \rho_{11}^{\mathrm{ss}} \\ \mathrm{Im} \, \rho_{12}^{\mathrm{ss}} \end{pmatrix} \end{aligned} \tag{6}$$

with

$$\mu \equiv \frac{1}{4} \left(16\Omega^2 - \Gamma^2 \right)^{1/2} \tag{7}$$

and the stationary state density matrix elements of the atomic interaction picture density matrix $\rho_{\rm AI}$ given by

$$\rho_{11}^{\rm ss} = \frac{\Gamma^2 + \Omega^2}{\Gamma^2 + 2\Omega^2} \, , \quad {\rm Im} \, \rho_{12}^{\rm ss} = \frac{\Gamma \Omega}{\Gamma^2 + 2\Omega^2} \, , \tag{8}$$

and

$$\mathrm{Re} \, \rho_{12}^{\rm ss} = 0 \, . \tag{9}$$

The atomic system under consideration possesses a stationary state only in the interaction picture. In the Schrödinger picture, its off-diagonal matrix elements become time-dependent. As a result we choose to remain in the interaction picture wherein the observable $\sigma_{\rm x}$, in which we are interested, becomes time-dependent.

C. The Mollow spectrum

As an alternative to the power spectrum considered here, laser-driven atomic two-level systems are often characterised by the so-called Mollow triplet of resonance fluorescence [17, 18]. Since the spectrum we consider has many similarities with Mollow's spectrum, we briefly summarise the main characteristics of the latter. It is defined as the Fourier transform of the correlation function

$$G_{\rm Mol}(\tau) \;=\; \langle \sigma^+(\tau) \sigma^-(0) \rangle_{\rm ss} \, . \tag{10}$$

Calculating this expression with the help of the master equations presented in the previous section yields [18]

$$\begin{aligned} S_{\rm Mol}(\omega) \;=\; & \frac{2\Omega^2}{\Gamma^2 + 2\Omega^2} \left[\frac{4\Gamma}{\Gamma^2 + 4\delta^2} \right. \\ & + 4 {\rm Re} \left(\frac{3\Gamma - 4{\rm i}(\delta + \mu)}{9\Gamma^2 + 16(\delta+\mu)^2} \, \beta_{\rm Mol}^+ \right) \\ & \left. + 4 {\rm Re} \left(\frac{3\Gamma - 4{\rm i}(\delta - \mu)}{9\Gamma^2 + 16(\delta-\mu)^2} \, \beta_{\rm Mol}^- \right) \right] \end{aligned} \tag{11}$$

with μ defined as in Eq. (7), the detuning δ given by

$$\delta \;\equiv\; \omega - \omega_0 \, , \tag{12}$$

and with

$$\beta_{\rm Mol}^{\pm} \;\equiv\; - \frac{\Gamma^2 - 2\Omega^2}{4(\Gamma^2 + 2\Omega^2)} \mp \frac{{\rm i}\Gamma}{16\mu} \left[1 - \frac{12\Omega^2}{\Gamma^2 + 2\Omega^2} \right] \, . \tag{13}$$

This spectrum has its maximum at the atomic transition frequency ω_0. In the case of sufficiently strong driving, two sidebands of equal height appear at $\omega = \omega_0 - \mu$ and $\omega = \omega_0 + \mu$. For weak driving, the sidebands vanish and there is only a single peak.

Unfortunately, there seems to be no clear operational definition of the power spectrum defined by the correlation function in Eq. (10). In order to relate this quantity to the properties of photons emitted by the atomic system one starts with the assumption that the atom-field system can be viewed as closed. One then solves for the dynamics of the electric field within the Heisenberg picture and employs the rotating wave approximation to simplify the result. In this context the rotating wave approximation constitutes the identification of the positive frequency component of the electric field (photon annihilation operator) with the atomic lowering operator. Similarly one identifies the negative frequency component (photon creation operator) with the atomic raising operator. Thus, within the rotating wave approximation the solutions of the equations of motion for the positive and negative frequency components of the electric field are effectively of the form $\mathbf{E}^{\mp}(t) \propto \sigma^{\pm}(t)$, with the result that Glauber's first order correlation function $\langle \mathbf{E}^-(t+\tau) \mathbf{E}^+(t) \rangle$ gives Eq. (10) when it is assumed that the atom is in the stationary state. The correlation function $\langle \mathbf{E}^-(t+\tau) \mathbf{E}^+(t) \rangle$ is *supposed* to furnish a measure of correlations in photon number at two distinct times [34], but its exact physical interpretation is not entirely clear [35]. Nevertheless its Fourier transform is usually taken as the *definition* of the fluorescence spectrum.

III. THE POWER SPECTRUM OF THE ELECTRIC FIELD

In classical physics, the power spectrum $S(\omega)$ of a signal $f(t)$ is defined as the modulus squared of its Fourier transform $\tilde{f}(\omega)$ which implies

$$S(\omega) \;\equiv\; |\tilde{f}(\omega)|^2 \, . \tag{14}$$

For a real signal $f(t)$, this equation yields

$$S(\omega) \;=\; \int_{-\infty}^{\infty} {\rm d}t \int_{-\infty}^{\infty} {\rm d}t' \, {\rm e}^{-{\rm i}\omega(t-t')} \, f(t) \, f(t') \, . \tag{15}$$

After substituting $\tau = t - t'$, one can easily see that the power spectrum of a signal is the Fourier transform of its two-time correlation function $G(\tau)$,

$$S(\omega) \;=\; \int_{-\infty}^{\infty} {\rm d}\tau \, {\rm e}^{-{\rm i}\omega\tau} \, G(\tau) \tag{16}$$

with $G(\tau)$ defined as

$$G(\tau) \;=\; \int_{-\infty}^{\infty} {\rm d}t \, f(t) \, f(t+\tau) \, . \tag{17}$$

In quantum physics, the correlation function $G(\tau)$ can reflect a very rich inner dynamics of the system under investigation. A measurement at time t changes the state of a quantum system in a non-trivial way. As a result, measurement outcomes can be highly correlated with previous measurements.

Let us now have a look at the power spectrum of the atomic dipole moment of a laser-driven atomic two-level system which is given by $f(t)$ in Eq. (1). If the signal $f(t)$ reaches a stationary state $f_{\rm ss}$, then $f(t) = f_{\rm ss}$ most of the time and $G(\tau)$ in Eq. (17) simplifies to $G(\tau) = f(0) \, f(\tau)$

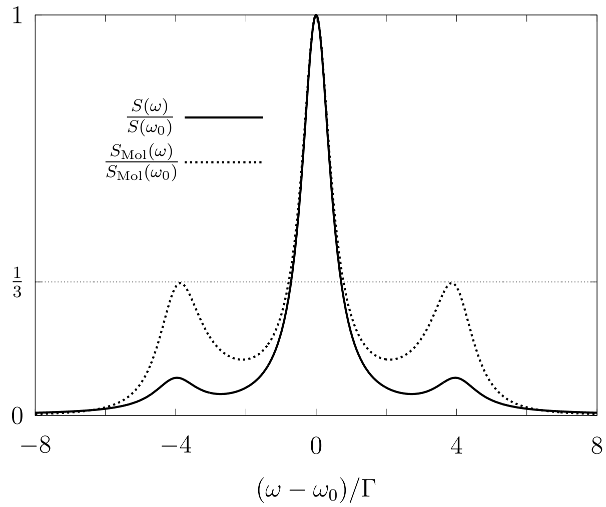

FIG. 1: The normalised power spectrum $S(\omega)/S(\omega_0)$ and the normalised Mollow triplet $S_{\rm Mol}(\omega)/S_{\rm Mol}(\omega_0)$ in Eqs. (24)–(13) as a function of ω for $\Gamma = 10^8$ Hz, $\omega_0 = 10^{15}$ Hz, and $\Omega = 4\,\Gamma$.

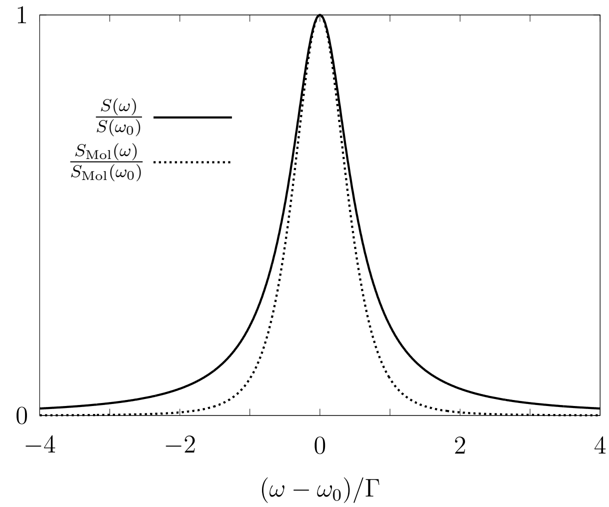

FIG. 2: The normalised power spectrum $S(\omega)/S(\omega_0)$ and the normalised Mollow triplet $S_{\rm Mol}(\omega)/S_{\rm Mol}(\omega_0)$ as a function of ω for the same Γ and ω_0 as in Fig. 1 but for $\Omega = 0.5\,\Gamma$.

if at $t = 0$ the system is in its stationary state. However, the signal we consider evolves constantly in time and therefore, we cannot calculate $G(\tau)$ via this simplification. We can, however exploit the fact that the system possesses a stationary state and calculate $G(\tau)$ using the interaction picture with respect to H_0 in Eq. (4). Within this interaction picture, the observable $\sigma_{\rm x}$ becomes time-dependent and equals

$$\sigma_{\rm x}(t) \;=\; \sigma^+\,{\rm e}^{-{\rm i}\omega_0 t} + \sigma^-\,{\rm e}^{{\rm i}\omega_0 t}\,. \tag{18}$$

The time-independent eigenvectors of this operator,

$$|\lambda_{1,2}(t)\rangle \;=\; \left(|1\rangle \pm {\rm e}^{{\rm i}\omega_0 t}\,|2\rangle\right)/\sqrt{2} \tag{19}$$

oscillate constantly in time. Moreover, one can show that its eigenvalues are given by

$$\lambda_{1,2} \;=\; \pm 1\,. \tag{20}$$

The equation implies that when detecting the electric field $\mathbf{E}(\mathbf{x})$, there are only two possible measurement outcomes. The laser-driven atomic system exerts a force of a certain size onto a test charge which points in one out of two opposite directions.

Using the notation in Eqs. (19) and (20) and normalising the correlation function $G(\tau)$ in Eq. (17) as usual, we find that $G(\tau)$ equals

$$\begin{aligned} G(\tau) \;=\; & \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}{\rm d}t \sum_{i,j=1,2}\lambda_i\lambda_j \\ & \times {\rm Tr}\left[\,I\!\!P_j(t+\tau)\,\mathcal{T}_\tau\left(I\!\!P_i(t)\rho_{\rm AI}(t)I\!\!P_i(t)\right)\right] \end{aligned} \tag{21}$$

with the superoperator $\mathcal{T}_\tau$ summarizing the atomic time evolution in Eqs. (5) and (6);

$$\mathcal{T}_\tau(\rho_{\rm AI}(t)) \;=\; \rho_{\rm AI}(t+\tau)\,, \tag{22}$$

and with $I\!\!P_i(t)$ being the projector onto the time-dependent state $|\lambda_i(t)\rangle$,

$$I\!\!P_i(t) \;=\; |\lambda_i(t)\rangle\langle\lambda_i(t)|\,, \tag{23}$$

In other words, $G(\tau)$ is the expectation value for the product of the results of two subsequent measurements of the atomic dipole moment $\sigma_{\rm x}$ – one taken at t and one taken at $t+\tau$. The state immediately after the first measurement is the eigenstate corresponding to the first measurement's outcome. It is independent of the initial state of the system (at $t = 0$) due to the averaging over an infinitely long time scale ($T \to \infty$). This implies $\rho_{\rm AI}(t) = \rho_{\rm AI}^{\rm ss}$ at almost all times t.

Taking this into account and evaluating Eq. (21) with the help of Eqs. (5)–(8), yields

$$\begin{aligned} G(\tau) \;=\; & \frac{1}{2}\left[{\rm e}^{-\Gamma\tau/2} + \left(\beta^+\,{\rm e}^{{\rm i}\mu\tau} + \beta^-\,{\rm e}^{-{\rm i}\mu\tau}\right){\rm e}^{-3\Gamma\tau/4}\right. \\ & \left. + \frac{4\Gamma^2\Omega^2}{(\Gamma^2+2\Omega^2)^2}\right]\cos\omega_0\tau \end{aligned} \tag{24}$$

with the coefficients $\beta_\pm$ given by

$$\beta^\pm \;\equiv\; \frac{\Gamma^4+4\Omega^4}{2(\Gamma^2+2\Omega^2)^2} \mp \frac{{\rm i}\Gamma}{8\mu}\left[1 - \frac{12\Gamma^2\Omega^2}{(\Gamma^2+2\Omega^2)^2}\right]. \tag{25}$$

Substituting this result into Eq. (16) and neglecting a sharp δ-peak due to the first term in Eq. (24), we finally obtain the power spectrum $S(\omega)$ of the electric field generated by a resonantly-driven atomic system,

$$\begin{aligned} S(\omega) \;=\; & \frac{2\Gamma}{\Gamma^2+4\delta^2} + 2{\rm Re}\left(\frac{3\Gamma-4{\rm i}(\delta+\mu)}{9\Gamma^2+16(\delta+\mu)^2}\,\beta^+\right) \\ & + 2{\rm Re}\left(\frac{3\Gamma-4{\rm i}(\delta-\mu)}{9\Gamma^2+16(\delta-\mu)^2}\,\beta^-\right)\,, \end{aligned} \tag{26}$$

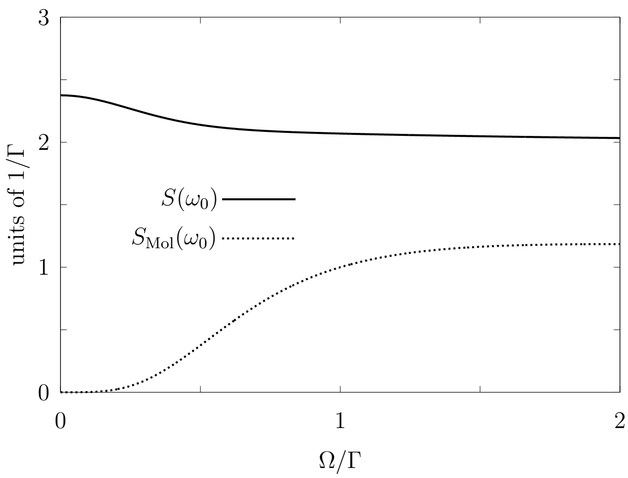

FIG. 3: The relative amplitudes of the Comparison of the unnormalised height of the maximum peak of $S(\omega)$ and $S_{\rm Mol}(\omega)$ for $\Gamma = 10^8$ Hz, $\omega_0 = 10^{15}$ Hz and $\Omega = 0.5\,\Gamma$.

where the frequencies μ and δ are as defined in Eqs. (7) and (12). This spectrum has some similarities with the Mollow spectrum $S_{\rm Mol}(\omega)$ in Eq. (11) but also certain differences.

Figs. 1 and 2 show the ω-dependence of S (solid lines) and $S_{\rm Mol}$ (dashed lines) for two different values of the laser Rabi frequency Ω. Here the amplitudes of both spectra have been normalised to unity. When Ω is relatively small there is only a single central peak. For sufficiently strong laser driving, both spectra exhibit three distinct peaks indicating both spectra contain similar information about the atomic dynamics. There are, however some differences. For example, the relative amplitudes of the sidebands of $S(\omega)$ are significantly smaller than the sidebands of Mollow's spectrum.

Another difference between $S(\omega)$ and $S_{\rm Mol}(\omega)$ is illustrated in Fig. 3. This figure shows the dependence of the height of the central peaks with the laser Rabi frequency Ω. The Mollow triplet is normalised such that $\int {\rm d}\omega S_{\rm Mol}(\omega)$ is a direct measure of the stationary state photon emission rate $I_{\rm ss}$ of the laser-driven atomic system [18, 19]. This means, its amplitude tends to zero, when Ω tends to zero. In contrast to this, $S(\omega)$ assumes its maximum when $\Omega \to 0$. This means the spectrum $S(\omega)$ is not a measure of the photon emission intensity of the atomic source. A closer look at Eqs. (19) and (20) shows that $\langle \mathbf{E}(\mathbf{x}) \rangle_t$ is non-zero, even when the atomic system is in its ground state. When brought sufficiently close, the atomic dipole moment is expected to exert a force on a test charge. When this force is measured, the atomic state changes accordingly either into $|\lambda_1(t)\rangle$ or into $|\lambda_2(t)\rangle$. We think that it would be interesting to observe this effect experimentally.

IV. CONCLUSIONS

This paper calculates the power spectrum $S(\omega)$ of the electric field $\langle \mathbf{E}(\mathbf{x}) \rangle_t$ generated by a laser-driven atomic two-level system as would be observed at a relative position $\mathbf{x}$. The detector should be placed a very small distance away from the atomic source which is in principle feasible experimentally using currently available technology [6–11]. The derived expression for $S(\omega)$ in Eq. (26) has some similarities with Mollow's resonance fluorescence spectrum [17–19] but there are also several differences. For sufficiently strong laser driving there is a central peak as well as two sidebands but their relative height is significantly reduced (cf. Figs. 1 and 2). Moreover, the amplitude of this spectrum has a different dependence on the laser Rabi frequency Ω (cf. Fig. 3). It assumes its maximum when Ω tends to zero.

The main result of this paper is to identify the electric field generated by a laser-driven atomic two-level system. Due to the presence of a photon-absorbing environment, there is on average never more than one photon in the radiation field [12–15]. As a consequence, the only non-zero contribution to $\langle \mathbf{E}(\mathbf{x}) \rangle_t$ comes from the atomic dipole moment $\mathbf{r}$. This quantity can be calculated analytically without any approximations. The predicted spectrum $S(\omega)$ contains a similar amount of information about the atomic system dynamics as Mollow's spectrum but can be calculated in a more direct way and constitutes an interesting alternative property. We hope that this paper provides new insight into the dynamics of quantum optical systems with spontaneous photon emission. The present discussion can be extended relatively easily to more complex systems.

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